LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2022**

17/18UMT3MC02 – VECTOR ANALYSIS AND ORDINARY DIFF. EQUATIONS

Date: 03-12-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL questions: **(10 × 2 = 20 Marks)**

1. When do you say a vector is solenoidal and irrotational?
2. If $= x^2y^3z^2$, find $\nabla\varphi$.
3. Show that $\vec{F} = (2xy + z^3)\vec{i} + x^2\,\vec{j} + 3xz^2\,\vec{k}$ is a conservative vector field.
4. State Stoke's theorem.
5. Evaluate $\int \vec{F}\,.\,d\vec{r}$ where $\vec{F} = x^2\vec{i} + y^2\vec{j}$ along the line y=x from A(0, 0) to B(1, 1).
6. Find the unit vector normal to the surface $\phi = xyz - 1$ at the point (1,1,1).
7. Solve $\frac{dy}{dx} = \frac{y+2}{x+3}$.
8. Find the general solution of $y = xp + \frac{\alpha}{p}$.
9. Find the complete integral of $(D^2 - 9)y = 0$.
10. Define Cauchy Euler equation.

PART – B

Answer any FIVE questions: **(5 × 8 = 40 Marks)**

11. Prove that for any vector $\vec{F}$, $\nabla \times (\nabla \times \vec{F}) = \nabla(\nabla.\vec{F}) - \nabla^2\vec{F}$.
12. Show that (a) $\nabla(1/r) = -\vec{r}/r^3$ (b) $\nabla f(r) = f'(r)\hat{r}$, where $r = x\vec{i} + y\vec{j} + z\vec{k}$ and $|\vec{r}| = \hat{r}$.
13. Evaluate $\iint_s \vec{F}.n\ ds$ where $\vec{F}.\vec{n} = z\vec{x} + x\vec{j} - y^2z\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 1$ included in the first octant between z = 0 and z = 2.
14. Verify Stoke's theorem for $A = xy\vec{i} + yz\vec{j} + xz\vec{k}$ taken over the triangular surface S in the plane $x + y + z = 1$ bounded by the planes $x = 0, y = 0, z = 0$.
15. By Green's theorem, find the value of $\int_c x^2ydx + ydy$ along the closed curve C formed by $y^2 = x$ and $y = x$ between (0,0) and (1,1).
16. Solve $xp^2 - 2yp + x = 0$.
17. Find the solution of $(D^2 - 4D + 3)y = e^{-x}sinx$.
18. Solve $3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$.

PART – C

Answer any TWO questions: **(2 × 20 = 40 Marks)**

19. a) Find the value of a if $A = (axy - z^2)\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 - axz)\vec{k}$ is irrotational. **(10)**

    b) Find the maximum value of the directional derivative of the function $\emptyset = 2x^2 + 3y^2 + 5z^2$ at the point $(1,1,-4)$. **(10)**

20. (a) Evaluate $\iint_S \vec{F}.n \, ds$ where $\vec{F} = z\vec{i} + y^2\vec{j} + yz\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5. **(10)**

    (b) Verify divergence theorem for $\vec{A} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over region bounded by the surfaces $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **(10)**

21. (a) Solve $y = xp + x(1 + p^2)^{\frac{1}{2}}$. **(10)**

    (b) Solve $(D^2 - 4D - 5)y = cosx + e^{-x}$. **(10)**

22. Solve $\frac{d^2y}{dx^2} + y = sec\, x$, using variation of parameters. **(20)**

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